Comparative Study of Multilayer Perceptron and Radial Basis Function Artificial Neural Networks for Rainfall-Runoff Modeling in a Watershed

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Received: 17.11.2017 | Revised: 22.12.2017 | Accepted: 26.12.2017

ABSTRACT
This paper compares the performance of two artificial neural network (ANN)- multilayer perceptron (MLP) and radial basis function (RBF) for modeling daily rainfall-runoff in a Himalayan watershed called Bino watershed situated at Almora and Pauri Garhwal districts of Uttarakhand, India. The time series monsoon data of rainfall and runoff between 2000 and 2009 were used to train and test the models. The best input combination was selected by gamma test (GT) technique. The performance of both the MLP and RBF neural network models were comprehensively evaluated in terms of indices viz. correlation coefficient (r), root mean square error (RMSE) and coefficient of efficiency (CE). The results of the study indicate that the choice of the network type has certainly an impact on the prediction accuracy of model. Both models performed satisfactorily for runoff predictions; however, the MLP model outperformed the RBF model. The r, RMSE, CE and $R^2$ values for the best MLP model during testing were determined to be 0.92, 0.96 (mm), 0.80 and 0.85, respectively. Results show that ANN models are useful tools for rainfall-runoff modeling the hydrologic response with good accuracy in the study watershed.

Key words: Artificial neural network, Multilayer perceptron, Radial basis function, Gamma test, Rainfall-Runoff Modelling

INTRODUCTION
Over the last few decades, a number of mathematical rainfall–runoff models have been developed to quantify and understand watershed-based hydrologic processes such as conceptual, physically based distributed models and black box models. In a rigorous theoretical sense, the physically based models can be considered a better choice. However, the significant data requirements of such models, coupled with longer time taken in modeling make them an unfavourable choice.9. Modeling of rainfall-runoff process is important for watershed management and water resources planning. However, it is extremely complex due to its non-linear, multi-dimensional and inter-relationships nature of underlying climatic and physiographic factors exhibiting both temporal and spatial variation.

Cite this article: Chanu S.N. and Kumar P., Comparative Study of Multilayer Perceptron and Radial Basis Function Artificial Neural Networks for Rainfall-Runoff Modeling in a Watershed, Int. J. Pure App. Biosci. 5(6): 1471-1481 (2017). doi: http://dx.doi.org/10.18782/2320-7051.6078

Available online at www.ijpab.com
DOI: http://dx.doi.org/10.18782/2320-7051.6078
ISSN: 2320 – 7051
In cases with high degree of uncertainty and complexity where it is difficult to consider every effective physical parameter, black box models like ANN can be used effectively overcoming the limitations of conceptual and physical based models. Nourani and Komasi\textsuperscript{28} stated that that black box models may produce more accurate results than physical based models. ANNs are artificial intelligence-based self adaptive computational tools that mimic the biological functional of a human brain. Many researchers have successfully utilized ANN for modeling rainfall–runoff phenomena\textsuperscript{2,3,7,8,11,13,14,20,25,29,36,37}. The two most commonly used ANN for modeling rainfall–runoff processes are the multilayer perceptron (MLP) and radial basis function neural network (RBFNN) models\textsuperscript{16}. Many studies have reported on comparisons of the two models in simulating rainfall–runoff processes\textsuperscript{5,16,24,31,34,40}. Generally both models are found to give satisfactory in modeling hydrologic processes and the performance of the model is reported to depend upon choice of network type and input variables used. These studies have recommended that many more evaluations are needed before establishing a clear choice of network. RBFNN is considered among the most popular tools for function approximation, classification problems, noisy interpolation and have been recently widely used in varieties of fields, such as non-linear control, speech processing and pattern recognition\textsuperscript{10,30}. Zounemat-Kermani et al.\textsuperscript{40} predicted daily watershed runoff comparing the performance of two different artificial neural networks (ANNs) viz. multilayer feed-forward neural network using Levenberg–Marquardt learning algorithm (LMFF) and radial basis function neural network (RBFNN). Nemati et al.\textsuperscript{24} simulated rainfall-runoff process using MLPNN and RBFNN and their performance was being compared general regression neural network (GRNN). Phuokoefhim et al.\textsuperscript{41}, compared the performance of gene expression programming (GEP) with two different neural networks MLPNN and RBFNN for development of multimodel systems. One of the important phases in modeling is identifying the best input combination of the network\textsuperscript{17}. Maier and Dandy\textsuperscript{18} have also mentioned that determining of adequate model inputs and development of suitable network architecture are key aspects requiring further attention. Selection of input combination in hand before running the model could save a lot of modeling time and help in giving the optimal result. Many researchers have reported the used of various methods for reducing the number of input variables such as principal component analysis (PCA), Gamma test (GT), forward selection (FS) etc. In this paper study gamma test (GT) has been used. Only a few studies have applied in the field of water resources management\textsuperscript{27} and it is a new technique. The GT was firstly reported by Stefansson et al.\textsuperscript{36}, Koncar\textsuperscript{15} and Agalbjörn et al.\textsuperscript{1} and later it was discussed and utilized by many scientists\textsuperscript{4,6,17,22,26,27,32,38}. Considering the above points, the objectives of this study was selected to develop and evaluate the ability of MLPNN and RBFNN models to predict runoff using GT as the best input combination selection technique in Bino watershed, India.

**MATERIALS AND METHODS**

**Study area and data**

The Bino watershed with a drainage area of 296.366 Km\textsuperscript{2} is situated in North-Eastern part of Ramganga catchment in middle and outer ranges of Himalayas between 79° 6’ 14.4” E to 79° 17’ 16.8” E longitude and 29° 47’ 6” N to 30° 02’9.6” N latitude in Almora and Pauri Garhwal districts of Uttarakhand, India (Fig. 1). The watershed has very undulating topography with mean length of 28.46 Km and 17.27 Km and irregular slopes varying from moderate to steep in valley areas on either sides of the Bino River. The climate of the watershed varies from Himalayan sub-tropical to sub-temperate with mean annual maximum and minimum air temperature of 30 °C to 18 °C, respectively. The daily mean temperature remains higher during the months of May and June and minimum in December and January. Based on the rainfall data for the years 2000 to 2009, the mean annual rainfall in the area is 687.53 (mm). Daily rainfall and runoff data of 10 years (2000-2009) were collected from Divisional Forest and Soil Conservation Office, Ranikhet, Uttarakhand, India.
Gamma test (GT)

Gamma test is one of the non-linear modeling and analysis tools that can investigate an underlying input-output relationship in a numerical data set as well as establishing a smooth model. GT estimates the minimum mean square error (MSE) that can be achieved when modeling the unseen data using any continuous non-linear models\(^2\). Suppose there exists a set of data observations as \{(x_i, y_i), 1 \leq i \leq M\} where the input vectors \(x_i \in \mathbb{R}^m\) are \(m\) dimensional vectors (with a record length of \(M\)) confined to some closed bounded set \(C \in \mathbb{R}^m\) and \(y_i \in \mathbb{R}\) is corresponding outputs scalar. If the underlying relationship between input-output can be expressed as:

\[
y = f(x_1, \ldots, x_m) + r
\]

where \(f\) is a smooth unknown function and \(r\) is a random variable representing noise. GT allows the variance of the noise variable \(r\) (Var(\(r\))) to be estimated, despite the fact that \(f\) is unknown. GT calculates model output variance that cannot be accounted by a smooth data model called Gamma statistic (\(\xi\)). GT is based on the \(k\)th \((1 \leq k \leq p)\) nearest neighbors \(x_{N[k]}\) for each vector \(x_i\) \((1 \leq i \leq M)\) and \(p\) is the number of near neighbors, typically \(p = 10^{12}\).

It can be derived from Delta function of the input vectors which calculates the mean squared distance of the \(k^{th}\) neighbor:

\[
\delta_M(k) = \frac{1}{M} \sum_{i=1}^{M} |x_{N[i,k]} - x_i|^2 : (1 \leq k \leq p)
\]

where \(|...|\) denotes Euclidean distance, and corresponding Gamma function output is given as:

\[
\gamma_M(k) = \frac{1}{2M} \sum_{i=1}^{M} |y_{N[i,k]} - y_i|^2 : (1 \leq k \leq p)
\]

where \(y_{N[i,k]}\) is the corresponding \(y\)-value for the \(k^{th}\) nearest neighbor of \(x_i\) in Eq. (2).

To compute \(r\), a least squares regression line which is fitted for \(p\) points \((\delta_M(k), \gamma_M(k))\) as:

\[
y = A\delta + r
\]

The intercept on the vertical this axis \((\delta = 0)\) is the \(r\) value as \(\gamma_M(k) \rightarrow \text{Var}(r)\) in probability as \(\gamma_M(k) \rightarrow 0\). Selecting the most important and influencing parameters of a nonlinear and unknown function is one of the most difficult tasks of Gamma test.

Fig. 1: Location map of Bino watershed
steps in model development. If \( n \) number of the input variables exists, the combination of \( 2^n - 1 \) would be among them and analyzing all these combinations consumes lots of time. Therefore, GT was used in the present study for selecting the best combination of the input variables and it was achieved through winGamma\textsuperscript{TM} software implementation\textsuperscript{6}.

**Multilayer perceptron neural network (MLPNN)**

MLP is one of the most popular ANN architecture used for hydrological modeling. Rumelhart *et al.*\textsuperscript{33} is considered to be first to introduce MLP with back propagation training algorithm for training of neural networks which considerably brought about significant growth in application of ANN in different fields. MLPNN is multilayer feed forward network typically trained with static back propagation and are made up of multiple layers of neurons. In this architecture, besides the input and the output layer, there is one or more than one intermediate layer(s) called hidden layer(s). Each layer is fully connected to the preceding layer by interconnection strengths or weights. A typical three layer MLP structure is shown in Fig. 2.

![Fig. 2: A three layer MLP structure](image)

**Radial basis function neural network (RBFNN)**

The RBFNN is a non-linear is a feed-forward hybrid type of artificial neural network typically containing a single hidden layer of processing elements (PEs) or nodes (Fig 3). The RBFNN is one of the most popular ANN architecture used for hydrological modeling. The RBFNN consists of only three layers (input, intermediate/radial basis and output layer). The number of neurons in input and output are problem dependent and hidden neurons are determined by network designer. The processing elements in the adjacent layers are exhaustively connected. The transformation process of the input-output through the neurons of the hidden layer is being achieved using the non-linear radial basis function. There are a number of radial basis functions that can be applied in hidden layer neurons, such as the Gaussian function, the multi-quadratic function and the inverse multi-quadratic functions\textsuperscript{19}. However, the most widely used radial basis function is the Gaussian function and it has been also used in the present study. The Gaussian function is symmetrical in nature having only the positive values in range 0 and 1.
Mathematical representation of RBFNN

To represent mathematically, referring to Fig 3, let the input layer of RBFNN consist of $n_0$ input nodes, hidden layer of $n$ nodes and one output layer node for general transformation of $P$ points in input space to one point in output space. The connections between the input and hidden layers are not weighted. The Gaussian functions (RBFs) which are used as transfer functions at the hidden nodes can be expressed as,

$$\varphi_j = \exp \left( - \frac{\|X^i - \mu_j\|^2}{2\sigma^2} \right)$$

(5)

where,

$X^i = n_0$ dimensional input vectors, $i = 1, 2...P$;

$\mu_j =$ mean (center) of the radial basis function for hidden node $j$, $j = 1, 2...n$;

$\sigma_j =$ standard deviation (spread) and control the smoothness of RBF, $j = 1, 2...n$;

$n =$ number of hidden nodes

$\varphi_j =$ radial basis function value

$\|X^i - \mu_j\| =$ Euclidean distance between the center of the radial basis function and input.

The output of network is the summation of linear weights between hidden and output layers and represented as,

$$Q_i = \sum_{k=1}^{n} \varphi_k w_k + w_0$$

(6)

where,

$Q_i =$ output values corresponding input vector $X^i$, $i = 1, 2...P$;

$w_k =$ connection weights;

$w_0 =$ bias term which make the estimation unbiased

Development of model

In this study, daily rainfall and runoff data of monsoon period (1st June to 30th September) for the period 2000-2009 were used for training and testing of MLP NN and RBF NN models. Out of this, 70 % of data (2000 to 2006) were used for training and remaining 30 % of data (2007 to 2009) were used for testing of developed models. Best input combination was selected using GT technique and these inputs were used to train MLP NN and RBFNN for simulating current day runoff. The MLPNN with both single and double hidden layers were trained using Levenberg–Marquardt as learning rule (which is an improved second order method for gradient) and hyperbolic tangent as transfer function. For RBFNN, various models were trained by unsupervised learning for estimation of centres and widths of RBF and supervised learning for calibration of weights in output layer. During the unsupervised learning, conscience full competitive rule with Euclidean metric was used with a maximum epochs of 100. The number of Gaussians was entered using the cluster centers field and numbers of cluster centers were selected by trial and error procedure and the output layer was trained with Levenberg–
Marquardt learning rule and hyperbolic tangent transfer function considering maximum epochs of 1000 and training threshold of 0.001. NeuroSolutions 5.0 software using designed and written by Curt Lefebvre & Jose Principe was used to run both MLPNN and RBFNN models. The network training was stopped as soon as the maximum number of epochs, which was predetermined at 1000, and training threshold of 0.001 were reached. The optimum network of both MLPNN and RBFNN were selected based on the performance indices that yield the minimum root mean square error (RMSE), maximum correlation coefficient (r) and coefficient of efficiency (CE).

Model performance evaluation

Three criteria RMSE, r and CE have been used to assess the goodness of fit performance of the models:

\[
RMSE = \sqrt{\frac{\sum_{j=1}^{n} (O_j-P_j)^2}{n}} \quad (7)
\]

\[
r = \frac{\sum_{j=1}^{n} ((O_j-\bar{O})(P_j-\bar{P}))}{\sqrt{\sum_{j=1}^{n} (O_j-\bar{O})^2 \sum_{j=1}^{n} (P_j-\bar{P})^2}} \quad (8)
\]

\[
CE = (1 - \frac{\text{residual variance}}{\text{initial variance}}) = (1 - \frac{\sum_{j=1}^{n} (O_j-P_j)^2}{\sum_{j=1}^{n} (O_j-\bar{O})^2}) \quad (9)
\]

where, j is an integer varying from 1 to n, Oj, Pj, \(\bar{O}\), \(\bar{P}\) and n are observed value, predicted value, mean of observed value, mean of predicted value and the number of observations respectively. The RMSE was applied to measure prediction accuracy that produces a positive value by squaring the errors. r is used as an indicator of degree of closeness between predicted and observed values. The coefficient of efficiency is used for comparing the relative performance of two approaches effectively and commonly assesses the predictive power of hydrological models\(^{23}\). Theoretically CE varies from \(-\infty\) and 1, with 1 being corresponding to perfect model.

RESULTS AND DISCUSSION

Gamma test

The current day rainfall (\(R_t\)) and previous days rainfall (\(R_{t-1}, R_{t-2}, ..., R_{t-n}\)) as well as previous days runoff (\(Q_{t-1}, Q_{t-2}, ..., Q_{t-n}\)), were used as inputs to simulate current day runoff (\(Q_t\)), where n is number of lags. The results GT is shown in Table 1. According to the principals of the GT, the combination with the minimum gamma value is the best combination for modeling and showed that the data with the provided combination has the possibility to achieve a better result\(^{32}\). It is observed from the Table 1, the minimum gamma value was found to be least for model no. 8 with r value of 0.0642. Therefore, the combination \(R_t, R_{t-1}, R_{t-2}, Q_{t-1}, Q_{t-2}\) was selected as the best input combination and optimum variables for developing MLPNN and RBFNN models for predicting daily runoff in Bino watershed.

![Table 1: Results of GT for determining the best combination out of the input variables for runoff modelling](image-url)
Comparison of MLPNN and RBFNN runoff models

MLPNN models with varying hidden neurons of both single and double hidden layers have been trained and tested to select the optimal architecture of the network. All together 20 models i.e. MLP1 to MLP 20 has been developed and out of these, 10 are single hidden layer neural networks i.e. MLP1 to MLP10 and rest are double hidden layer neural networks. The results of performance evaluation indices values from the rainfall-runoff modeling of MLPNN models during training and testing found that the r values vary from 0.81 to 0.95 during training and 0.62 to 0.92 during testing. The RMSE values vary from 1.27 to 3.24 (mm) and 0.96 to 1.96 (mm) during training and testing, respectively. While the values of CE during training and testing varied from 0.19 to 0.88 and 0.16 to 0.80, respectively. It is also obtained that out of the 20 models developed, MLP7 (5-8-1) was found to be the best as compared to other networks based on the performance criteria. Among the double hidden neuron networks developed, MLP19 (5-10-11-1) was found to perform good. The r, RMSE and CE for MLP7 model during training period are found to be 0.95, 1.27 (mm) and 0.88, respectively and 0.92, 0.96 (mm) and 0.80, respectively are their respective values during testing. Out of the 11 RBFNN models (RBF1- RBF11) developed, the model RBF7 with number of cluster center 13 is found to be the best as compared to other models. The values of r, RMSE and CE for the best performing (RBF7) model during training and testing were found to be 0.93, 1.58 mm, 0.82 and 0.89, 1.27 mm and 0.65, respectively. Generally, the developed models were found to perform well during training; however, there is slight variation in performance during testing. This may be due to more number of data sets during training and therefore, the models were able map the inner lying relationship.

Qualitative performance of best developed models of both MLPNN and RBFNN were evaluated by comparing observed and predicted values of daily runoff in the form of time series and scatter plot during testing for the selected networks as shown in Figs. 4 and 5. It can be observed from these figures that the observed and predicted runoffs are in almost close agreement although there are under and over predictions in some data points during testing. During testing period, MLPNN model could predict better as compared to RBFNN model. This is clear from scatter plots with $R^2$ values 0.85 for MLPNN while for RBFNN, $R^2$ value was found to be 0.77 during testing period.
Fig. 4: Comparison of observed and predicted runoff and their corresponding scatter plot during testing period for MLP7 (5-8-1) model

\[ y = 0.7294x + 0.9764 \]
\[ R^2 = 0.85 \]

Fig. 5 Comparison of observed and predicted runoff and their corresponding scatter plot during testing period for RBF7 (5-13-1) model

\[ y = 0.8337x + 1.1009 \]
\[ R^2 = 0.77 \]
CONCLUSIONS
Based on the result of Gamma test (GT), the current day rainfall as well as previous two days consecutive rainfall and runoff of consecutive two previous days were found to be the best combination of input variables for daily rainfall-runoff modeling using MLPNN and RBFNN. The results of the study showed that this technique can be effectively applied prior to actual hydrological modeling saving lots of modeling time. Although RBFNN are faster in speed in training the networks during training, comparison of selected runoff models during testing found that the predictive performance of MLPNN is better than RBFNN runoff models for simulation of runoff in Bino watershed. The results also showed that both the ANN models could be successfully applied for daily rainfall-runoff modeling in Bino watershed, Uttarakhand.

Acknowledgements
The authors are very much thankful to Divisional Forest and Soil Conservation Office, Ranikhet, Uttarakhand, India for providing required data for the study and the first author is thankful to the Department of Science and Technology (DST), Ministry of Science and Technology, New Delhi for the financial support in the form of INSPIRE Fellowship rendered during her doctoral degree.

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